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Linkage of the Study of Learning Theories and Modern Algebra

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Abstract

Advanced mathematics influences the thinking of individuals in both domains. We conjecture that strengthened understanding of mathematics and learning theory will be an outcome of the reflection promoted by this parallel study, and that the deeper insights will contribute. The initial focus of the project is on Topics in Modern Algebra and on Theories for the Learning of Mathematics. We plan to adapt this approach to other undergraduate mathematics content areas. The learning theory course focuses most heavily on constructivist theories, though it also examines sociocultural and historical perspectives.

Keywords- Linkage, Learning, Content, Project, Theory Etc.

Introduction-

The study of topics in Modern algebra focuses on standard material that is found in many advanced undergraduate linear algebra courses mathematics from an advanced perspective. This paper reports on the results of two studies piloting the implementation of this approach, as well as the plans for implementation of dual courses Modern Algebra and the Learning of Modern Algebra. This project investigates three questions about the coordinated study of modern algebra and learning theories in mathematics: Do participants reflect upon and evaluate their own learning in terms of their study of learning theories? Do participants connect what they study about linear algebra or the learning theories to their planned mathematics content or pedagogy for their own high school mathematics teaching? Research at the National Center for Research in Teacher Education found that teachers who majored in the subject they taught often were not able to explain fundamental concepts in their discipline more clearly than other teachers. (McDiarmid & Wilson, 1991, p.i).

Their investigations led to the conclusion that “Teachers need explicit disciplinary focus, but few positive results can be expected by merely requiring teachers to major in an academic subject. knowledge organized for teaching deep understanding of the subject; awareness of conceptual barriers to learning; and knowledge of the historical, cultural, and scientific roots of mathematical ideas and techniques (Ma, 1999) The APOS framework utilizes qualitative methods for research and is based on a specific theoretical perspective that has been developed through attempts to understand the ideas of Piaget and Piaget The approach has three components.

It begins with an initial theoretical analysis of what it means to understand a concept and how that understanding could be constructed by the learner. The three components are cycled and both the theory and instructional treatments are revised as needed. This project supports an instructional approach that assists in reflective abstraction, helping to shape the mathematical mental structures needed for building Modern algebra concepts. The study of mathematics will be at a much higher, deeper or more conceptual level than is common for undergraduate linear algebra. The project incorporates a range of pedagogical activities including group activities, open writing, projects, and technology.

Methodology -

While listed as separate courses, our vision is that the mathematics content and the theory of learning courses are closely integrated, to the extent of shared time rather than strictly delimited schedules. The courses are co-taught by a mathematics faculty member and an education faculty member so that both professors participate in both the content and the learning theory courses. The primary goal is to study important mathematics—with some clear ties to high school content and teaching—in a way that leads to a deeper understanding into how it is learned. The curriculum and classroom organization is designed to provide rich opportunities for students to use and integrate linear algebra concepts with related topics from secondary mathematics.

The instructional practices used in these courses model approaches that reflect the learning theories being studied. Discussion is critical to success in this mode of instruction; it provides a rich opportunity to raise questions, share insights, clarify understanding, and express confusion. It provides a more natural setting to negotiate meaning and understanding with greater personal involvement; interactions can be lively and intense.. Parallel Courses in Modern Algebra and Learning of Modern Algebra Linear algebra was offered as a 3-credit dual-listed upper level undergraduate and graduate course required of mathematics majors. Twenty-five students, including three graduate students, enrolled in the course. Students who enrolled in the regular linear algebra course were invited to participate in a special two-credit seminar on learning theories in mathematics education.

Results-

For these initial pilot studies, we reviewed a final assessment from the learning theory and all the materials collected. Our three research questions were: Do participants make any connections between their study of linear algebra content and their study of learning theories? Do participants reflect upon and evaluate their own learning in terms of their study of learning theories? We found evidence of all forms of connections in the data.

However, the strongest evidence in the participants' responses was the evaluation of their own learning and the growth of understanding from the mix of activities they worked through. They also demonstrated in their writing and discussions reflective thinking about their learning and how they could apply what they had learned back to their own mathematics classrooms. Finally, while they used the language of APOS, for most there was very little clear connection between their analysis of the content of modern algebra and the theory to examine the theories and their own learning.

The weekend pilot study participants had more difficulty in understanding and applying APOS Theory in terms of the linear algebra content that they had learned. The following discussion of the three research questions uses analysis of students' work to help illustrate how the coordinated study of learning theory and modern algebra seemed to influence student thinking. For example, during one lesson in the learning theory course one participant asked a question about calculating the determinant of a 4x4 matrix, which had been assigned as an exploration. The question was specifically triggered by a student who was trying to make sense of the computation described in the textbook in which the author stated that the determinant of a 4x4 matrix is equal to the determinant of a 3x3 matrix plus or minus the product of an element and the determinant of a 3x3 matrix. This equivalence was due to a cofactor simplification expanding on a column with just one non-zero entry. The simplification was not explicitly explained at this point in the text's narrative. During a lengthy class discussion, students came to see the connections between determinants of matrices with different dimensions.

Discussion-

The discussion then turned to how APOS theory could be applied to this situation. Participants described their initial understanding as being at the action level; they were only able to compute determinants of specific matrices. When they came to understand the example given in the text, they suggested this required at least a process level understanding to recognize the simplification that had initially triggered the discussion. The students suggested that this recognition required an object level understanding of determinant. The APOS learning theory ideas helped the students recognize the increasing sophistication of their understanding of the computation of determinants, from an initial stage when they could only follow a computational algorithm for a specific dimension, to an understanding that gave them confidence that they could devise a strategy to compute any determinant, even one with variable rather than numeric elements. Participants recognized that they had gained a more sophisticated understanding of these computations by making sense for themselves of written material, discussing the ideas with fellow students and the instructor, and then reflecting on and describing their own thought processes. Conclusion- They believed that the understanding they had achieved could not come from direct instruction alone. Instead, their students—like they themselves—would need time and instructional experiences that also allowed and encouraged them to wrestle with mathematical ideas and generate their own explanations.

while interacting with other students and the teacher. Students in the seminar were asked for written responses to questions during the final course meeting. Two questions referred directly to APOS: (a) Describe the components of the APOS theory of learning, and (b) Choose a concept from Modern Algebra, then give a genetic decomposition of the concept according to APOS. The following genetic decomposition of modern algebra The other two students gave written examples of genetic decompositions that more superficially described the components of the APOS framework. Since students were not interviewed about their papers, we are unable to say more about their understanding of and ability to apply APOS theory to specific topics in linear algebra based on their final written assessment.

Conclusion-

The major themes that teachers raised across their discussions included the role of visualization in helping them to develop multiple representations as well as making connections between representations. They also included that the activities they were engaged in promoted higher- level thinking and sense-making in them. For example, in a discussion between the two concept maps about vector, Ellen1 wrote: In the first map we connected other concepts directly to vectors. We basically showed some ideas of what vectors represent.

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